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Aharoni and Berger conjectured [1] that every bipartite graph which is the union of n matchings of size $n+1$ contains a rainbow matching of size n. This conjecture is related to several old conjectures of Ryser, Brualdi, and Stein about transversals in Latin squares. There have been many recent partial results about the Aharoni-Berger Conjecture. When the matchings have size $2n$ then it is easy to see that the conclusion of the conjecture is true (by greedily choosing disjoint edges one at a time). Aharoni, Charbit, and Howard [2] proved that matchings of size $7n/4$ are sufficient to guarantee a rainbow matching of size n. Kotlar and Ziv [5] improved this to $5n/3$. Clemens and Ehrenmüller [3] further improved this to $3n/2 + o(n)$.

When the matchings are all edge-disjoint and perfect, then the best result follows from a theorem of Häggkvist and Johansson [4] which implies the conjecture when the matchings have size at least $n + o(n)$. In fact, Häggkvist and Johansson proved much more—they showed that in a bipartite graph consisting of edge-disjoint, perfect matchings each of size at least $n + o(n)$, it is possible to decompose all the edges into rainbow matchings of size n. Their proof is by a probabilistic argument, using a "random greedy process" to construct the matchings.

This seminar will be about a proof of the conjecture in the case when matchings have size $n + o(n)$ and are all edge-disjoint (but not necessarily perfect) [6]. The proof is algorithmic and so gives an alternative proof of the Häggkvist-Johansson Theorem. The proof involves studying connectedness in coloured, directed graphs. The notion of connectedness that is introduced is new, and perhaps of independent interest.

References

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