

The Local Action Lemma

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One of the main tools in probabilistic combinatorics is the *Lovász Local Lemma* (the LLL for short). Roughly speaking, it asserts that if the random events in a certain family are mostly independent and their marginal probabilities are somewhat small, then with positive probability none of them happen. The LLL is especially powerful, since the probability of avoiding all the events in the collection, although positive, can be exponentially small. A breakthrough result by Moser and Tardos states that, under the conditions only mildly more restrictive than those of the LLL, there exists an effective randomized algorithm that finds a point in the probability space avoiding all the events in the family. To estimate the running time of this algorithm, Moser and Tardos developed a new technique called the *entropy compression method*. It was discovered lately (and somewhat unexpectedly) that applying this method to particular combinatorial problems directly can give better results than using the LLL. We provide a purely probabilistic statement that generalizes the LLL and implies the new combinatorial results obtained using the entropy compression method.