Representing Random Permutations as the Product of Two Involutions

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Abstract

An involution is a permutation that is its own inverse. Given a permutation σ of $[n]$, let $\mathbf{N}_n(\sigma)$ denote the number of ways to write σ as a product of two involutions of $[n]$. If we endow the symmetric groups S_n with uniform probability measures, then the random variables $\mathbf{N}_n$ are asymptotically lognormal.

The proof is based upon the observation that, for most permutations σ, $\mathbf{N}_n(\sigma)$ can be well approximated by $\mathbf{B}_n(\sigma)$, the product of the cycle lengths of σ. Asymptotic lognormality of $\mathbf{N}_n$ can therefore be deduced from Erdős and Turán's theorem that $\mathbf{B}_n$ itself is asymptotically lognormal.